Published by Institute of Physics Publishing for SISSA/ISAS
Received: *February 06, 2002*
Accepted: *March 13, 2002*

The quark Regge trajectory at two loops

Andrey V. Bogdan and Victor S. Fadin

Budker Institute for Nuclear Physics, Novosibirsk State University
630090 Novosibirsk, Russia
E-mail: bezov@ngs.ru, V.S.Fadin@inp.nsk.su

Vittorio Del Duca

Istituto Nazionale di Fisica Nucleare, Sez. di Torino
via P. Giuria, 1 - 10125 Torino, Italy
E-mail: delduca@to.infn.it

E.W. Nigel Glover

Institute for Particle Physics Phenomenology, University of Durham
Durham, DH1 3LE, UK
E-mail: E.W.N.Glover@durham.ac.uk

Abstract: The hypothesis of the quark Reggeization is tested by taking the high-energy limit of the two-loop amplitude for quark-gluon scattering which was recently calculated. The limit is compatible with the Reggeization in the leading and the next-to-leading orders and allows the determination of the quark trajectory in the two-loop approximation. The trajectory is presented as an expansion in powers of $(D-4)$ for the space-time dimension D tending to the physical value $D=4$.

Keywords: QCD, NLO Computations, Hadronic Colliders.

Contents

1. Introduction

In the limit of large center of mass energy $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$ the most appropriate approach for the description of the scattering amplitudes is given by the theory of the complex angular momenta or Gribov-Regge theory. One of the remarkable properties of QCD is the Reggeization of its elementary particles. Unlike QED, where the electron Reggeizes [1, 2], but the photon remains elementary [3], in QCD both the gluon [4]–[10] and quark [11] Reggeize. We use here the notion "Reggeization" in the strong sense [12], that means not only the existence of a Reggeon with corresponding quantum numbers (including signature) and trajectory, but as well the dominance of the Reggeon contribution to the amplitudes of the processes with these quantum numbers in each order of perturbation theory. For example, parton-parton scattering amplitudes in QCD are dominated by gluon exchange in the crossed channel. The Reggeized gluon is a colour octet state of negative signature in the t-channel, i.e. odd under $s \leftrightarrow u$ exchange, and to leading logarithmic (LL) accuracy in $\ln(s/|t|)$ the virtual radiative corrections to the parton-parton scattering amplitude with the colour octet state and negative signature can be obtained, to all orders in α_S, by the replacement [8]

$$\frac{s}{t} \longrightarrow \frac{1}{2}\left[\left(\frac{-s}{-t}\right)^{j_G(t)} - \left(\frac{s}{-t}\right)^{j_G(t)}\right], \tag{1.1}$$

where $j_G(t) \equiv \omega(t) + 1$ is called the Regge trajectory of the gluon.

The property of the Reggeization is very important for high energy QCD. The BFKL equation [7]–[10] for the resummation of the leading logarithmic radiative corrections to scattering amplitudes for processes with gluon exchanges in the t-channel is based on the gluon Reggeization. The Pomeron, which determines the high energy behaviour of cross sections, and the Odderon, responsible for the difference of particle and antiparticle cross sections, appears in QCD as a compound state of two and three Reggeized gluons, respectively. Similarly colorless objects constructed from Reggeized quarks and antiquarks should also be relevant to the phenomenological description of processes involving the exchange of quantum numbers.

The basic parameters of the Reggeons are their trajectories and the interaction vertices. To leading logarithmic accuracy, $\omega(t)$ is related to a one-loop transverse-momentum integration. In dimensional regularization in $D = 4 + 2\epsilon$ dimensions this can be written as

$$\omega(t) = g_S^2 \left(\frac{\mu^2}{-t}\right)^{-\epsilon} c_\Gamma\, \omega^{(1)} + \mathcal{O}(g_S^4)\,, \tag{1.2}$$

with

$$\omega^{(1)} = -C_A \frac{2}{\epsilon}\,, \qquad c_\Gamma = \frac{1}{(4\pi)^{2+\epsilon}} \frac{\Gamma(1-\epsilon)\,\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\,, \tag{1.3}$$

and $C_A = N$. The resummation of the real and virtual radiative corrections to parton-parton scattering amplitudes with gluon exchange in the crossed channel is related by unitarity to the imaginary part of the elastic amplitudes with all possible colour exchanges in the crossed channel. The radiative corrections to these amplitudes are resummed through the BFKL equation i.e. a two-dimensional integral equation which describes the interaction of two Reggeized gluons in the crossed channel.

The integral equation is obtained to LL by computing the one-loop corrections to the gluon exchange in the t channel. They are formed by a real correction: the emission of a gluon along the ladder [6] and a virtual correction: the one-loop Regge trajectory, eq. (1.1). The BFKL equation is then obtained by iterating recursively these one-loop corrections to all orders in α_s, to LL accuracy.

In recent years, the BFKL equation has been improved to next-to-leading logarithmic (NLL) accuracy [13, 14, 15]. A necessary ingredient has been the calculation of the two-loop Regge trajectory of the gluon [16]–[19]. Recently the gluon Regge trajectory was re-evaluated [20], in a completely independent way, by taking the high energy limit of the two-loop amplitudes for parton-parton scattering with gluon exchanges in the t-channel. The validity of the gluon Reggeization to NLL was confirmed and full agreement with previous results was found.

So far attention has focussed mainly on processes dominated by Reggeized gluon exchange. However, let us consider a scattering process with fermion exchange, namely quark-gluon scattering, which proceeds via the exchange of a quark in the crossed channel, and let us take the limit $s \gg |u|$. Since in the center-of-mass frame of a two-particle scattering $u = -s(1+\cos\theta)/2$, the limit $s \gg |u|$ corresponds to backward scattering. By crossing symmetry, this is equivalent to quark pair annihilation into two gluons at small angles (in which case the roles of s and t are exchanged). The contribution from the exchange of a colour triplet state and of positive signature i.e. even under $s \leftrightarrow t$ exchange Reggeises, so that the virtual radiative corrections to the quark-gluon scattering amplitude with the colour triplet state and positive signature in the limit $s \gg |u|$ can be obtained, to all orders in α_S and to LL accuracy in $\ln(s/|u|)$, by the replacement [11]

$$\sqrt{\frac{s}{-u}} \longrightarrow \frac{1}{2}\sqrt{\frac{s}{-u}}\left[\left(\frac{s}{-u}\right)^{\delta_T(u)} + \left(\frac{-s}{-u}\right)^{\delta_T(u)}\right], \tag{1.4}$$

where the quark Regge trajectory is $j_Q(u) = \delta_T(u) + 1/2$. $\delta_T(u)$ is related to a one-loop transverse-momentum integration, which, for massless quarks and up to replacing the colour factor C_A with C_F, is the same as for the gluon trajectory. Thus in dimensional regularization $\delta_T(u)$ can be written as,

$$\delta_T(u) = -g_S^2\, C_F\, \frac{2}{\epsilon}\left(\frac{\mu^2}{-u}\right)^{-\epsilon} c_\Gamma\,, \tag{1.5}$$

with $C_F = (N^2-1)/(2N)$, and c_Γ given in eq. (1.3). This is similar to what happens in QED, where the electron also Reggeizes [1, 2]. Analogously to the gluon case, an equation was derived [11] to resum the real and virtual radiative corrections to exchanges with arbitrary colour state and signature. However, unlike from the BFKL equation for gluon exchange, that equation has not been solved.

In this paper we explicitly take the high-energy limit of the one-loop [21, 22] and of the two-loop amplitudes for quark-gluon scattering [23]. This allows us to test the validity of the quark Reggeization and to calculate the two-loop Regge trajectory of the quark.

Our paper is organised as follows. In section 2, we discuss the general structure of the quark-gluon scattering amplitudes, we evaluate them at tree and one-loop level in the limit $s \gg |u|$ and decompose them according to the irreducible colour representations exchanged in the u channel. In section 3 we discuss the Regge ansatz for resumming the LL and NLL and evaluate the leading and next-to-leading order corrections for the interference of the tree-amplitude with the Reggeized ansatz in terms of the gluon trajectory and the impact factors. Section 4 is devoted to the analysis of the one- and two-loop Feynman diagram calculations in the high energy limit and to the extraction of the LL, NLL and NNLL behaviours. We make a detailed comparison of the two approaches and show that the two approaches are compatible at the LL and NLL level. This allows the determination of the quark trajectory to two-loop order which we give as an expansion in powers of $(D-4)$ for the space-time dimension D tending to the physical value $D=4$. Finally, our findings are summarized in section 5.

2. The structure of the quark-gluon scattering amplitude

Let the incoming quark and gluon have momenta p_a and p_b, respectively, and the outgoing quark and gluon have momenta $p_{a'}$ and $p_{b'}$, respectively.

The most general possible colour decomposition for the amplitude $\mathcal{M}$ is

$$\mathcal{M} = 2(T^b T^{b'})_{a'a}\,\mathcal{A} + 2(T^{b'}T^b)_{a'a}\,\mathcal{B} + \delta^{bb'}\,\delta_{a'a}\,\mathcal{C}\,, \tag{2.1}$$

where b, b' are the colours of the two gluons and a, a' are the colours of the quarks. The factor 2 in eq. (2.1) is due to our choice for the normalization of the fundamental representation matrices, i.e. $\mathrm{tr}(T^aT^b) = \delta^{ab}/2$. The functions $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are colour-stripped sub-amplitudes where we have suppressed all dependence on the particle polarisations and momenta. At lowest order, for general kinematics, only the first two colour structures are present. They are related by $t \leftrightarrow u$ exchange, thus in the $s \gg |u|$ limit only one of them contributes. The third colour structure appears first at one-loop order and is symmetric under u and t exchange.

2.1 Quark-gluon amplitudes at tree and one-loop level

In the high-energy limit $s \gg |u|$, quark-gluon scattering (or any other crossing symmetry related process) is dominated by quark exchange. In this limit, at tree level accuracy only the configuration for which the outgoing gluon has the same helicity as the incoming quark will contribute. This is equivalent to say that in the limit $s \gg |u|$ helicity is conserved in the s channel. Thus, once the helicity along the quark line is fixed, of the two gluon helicity configurations allowed at tree level, only one dominates. The only other leading helicity configuration is the one obtained from this by parity, which flips the helicity of all the particles in the scattering.

Using the colour decomposition (2.1) in the helicity formalism [24], we note that there is only one independent colour-stripped tree sub-amplitude, the Parke-Taylor sub-amplitude [25]. Using the spinor products in the limit $s \gg |u|$ (which can be easily derived from the amplitudes listed in the appendix of ref. [26] in the limit $s \gg |t|$) in the sub-amplitude for the leading helicity configuration, we obtain the

$$\mathcal{A}^{(0)}(p_a^-, p_b^-; p_{a'}^-, p_{b'}^-) = -i\, g_S^2 \sqrt{\frac{s}{-u}} \left(\frac{p_{a'_\perp}}{|p_{a'_\perp}|} \right)^2 , \tag{2.2}$$

where complex transverse coordinates $p_\perp = p^x + ip^y$ have been used, and the superscipts in the argument on the left-hand side label the parton helicities.[1] Using eq. (2.1), the amplitude for quark-gluon scattering $q_a\, g_b \to q_{a'}\, g_{b'}$ for a generic tree-level helicity configuration may be written as,

$$\begin{aligned}\mathcal{M}^{(0)}(p_a^{\nu_q}, p_b^{\nu_g}; p_{a'}^{\nu_q}, p_{b'}^{\nu_{g'}}) &= -2i \left[g_S\, (T^b)_{a'i} C_{qg}^{(0)}(p_a^{\nu_q}, p_{b'}^{\nu_{g'}}) \right] \times \\ &\quad \times \sqrt{\frac{s}{-u}} \left[g_S\, (T^{b'})_{i\,a}\, C_{gq}^{(0)}(p_b^{\nu_g}, p_{a'}^{\nu_q}) \right] ,\end{aligned} \tag{2.3}$$

there the ν's denote the parton helicities, and we have explicitly enforced helicity conservation along a massless fermion line. From eq. (2.2), the tree-level coefficient function $C^{(0)}$ is,

$$C_{q\,g}^{(0)}(p_a^-, p_{b'}^-) = C_{g\,q}^{(0)}(p_b^-, p_{a'}^-) = \frac{p_{a'_\perp}}{|p_{a'_\perp}|} , \tag{2.4}$$

while for $s \gg |u|$ the unequal helicity coefficient functions of type $C^{(0)}(\pm\mp)$ are subleading. Squaring and summing eq. (2.3) over helicity and colour, we obtain,

$$\sum_{\mathrm{hel,\,col}} |\mathcal{M}^{(0)}|^2 = 8\, C_F^2\, N_c\, g_S^4\, \frac{s}{-u} = \frac{128}{3}\, g_S^4\, \frac{s}{-u} , \tag{2.5}$$

in agreement with the $s \gg |u|$ limit of the squared tree quark-gluon amplitudes.

[1] Note that, contrary to the conventions of the helicity formalism where all particles are taken as outgoing, here we label momenta and helicities as in the physical scattering.

In ref. [22], the coefficients in the colour decomposition (2.1) of the one-loop amplitude for the quark-gluon scattering have been calculated in the 't Hooft-Veltman and in the dimensional reduction infrared schemes. Using the results of ref. [22], the sub-amplitude of eq. (2.2) and the tree amplitude of eq. (2.3), we can write the unrenormalised one-loop amplitude in the helicity configuration of eq. (2.2), in the 't Hooft-Veltman scheme and in the limit $s \gg |u|$ as,

$$\begin{aligned}
&\mathcal{M}^{(1)}(p_a^-, p_b^-; p_{a'}^-, p_{b'}^-) = \\
&= \tilde{g}_S^2(u) \mathcal{M}^{(0)}(p_a^-, p_b^-; p_{a'}^-, p_{b'}^-) \left[\frac{2C_F}{-\epsilon} \ln\left(\frac{s}{-u}\right) - \frac{2(C_A + C_F)}{\epsilon^2} + \frac{3C_F}{\epsilon} + (\pi^2 - 7) C_F \right] + \\
&\quad + \tilde{g}_S^2(u)\, \delta^{bb'} \delta_{a'a}\, \mathcal{A}^{(0)}(p_a^-, p_b^-; p_{a'}^-, p_{b'}^-) \frac{2}{\epsilon} \ln\left(\frac{-s}{-t}\right) + \mathcal{O}(\epsilon)\,, \qquad (2.6)
\end{aligned}$$

to leading accuracy in s/u. Note that on the right-hand side the first term, which is proportional to the tree amplitude eq. (2.3), is real. The second term contains the new colour structure introduced at the one-loop level and is proportional to the tree sub-amplitude (2.2). It is purely imaginary, since to this accuracy $\ln(-s/-t) \simeq -i\pi$, where we used the usual prescription $\ln(-s) = \ln(s) - i\pi$, for $s > 0$. In eq. (2.6) and further in our paper we have rescaled the coupling as

$$\tilde{g}_S^2(u) = g_S^2 c_\Gamma \left(\frac{\mu^2}{-u}\right)^{-\epsilon}. \qquad (2.7)$$

In the one-loop amplitude helicity is not conserved in the s channel, thus the unequal helicity coefficient functions of type $C^{(1)}(\pm\mp)$ are no longer subleading. In fact, from ref. [22] we obtain

$$\mathcal{M}^{(1)}(p_a^-, p_b^\mp; p_{a'}^-, p_{b'}^\pm) = 2i \frac{g_S^4}{(4\pi)^2} (C_A - C_F) \sqrt{\frac{s}{-u}} (T^b T^{b'})_{a'a}\,, \qquad (2.8)$$

which is finite and can be written in the form of eq. (2.3) by replacing the one of the helicity conserving coefficient functions of type $C^{(0)}$ with the helicity violating coefficient function of type $C^{(1)}$ where

$$C_{q\,g}^{(1)}(p_a^-, p_{b'}^+) = C_{gq}^{(1)}(p_b^+, p_{a'}^-) = -\frac{g_S^2}{(4\pi)^2} (C_A - C_F) \frac{|p_{a'_\perp}|}{p_{a'_\perp}}\,. \qquad (2.9)$$

Note that in the calculation of a production rate the coefficient function of type $C^{(1)}(\pm\mp)$ appear only in the square of a one-loop amplitude.

Taking the interference of the one-loop amplitude (2.6) with the tree amplitude, (2.3), and summing over helicity and colour of initial and final states, we obtain,

$$\begin{aligned}
\sum_{\mathrm{hel,col}} \left(\mathcal{M}^{(1)} \mathcal{M}^{(0)*} \right) &= \sum_{\mathrm{hel,col}} |\mathcal{M}^{(0)}|^2\, \tilde{g}_S^2(u) \left[\frac{2C_F}{-\epsilon} \ln\left(\frac{s}{-u}\right) - \frac{2(C_A + C_F)}{\epsilon^2} + \frac{3C_F}{\epsilon} + \right. \\
&\quad \left. + (\pi^2 - 7) C_F - \frac{i\pi}{\epsilon} \frac{1}{C_F} \right] + \mathcal{O}(\epsilon). \qquad (2.10)
\end{aligned}$$

2.2 The colour structure in the u-channel

In order to gain more insight in how the amplitudes in the colour decomposition (2.1) are related to the exchange of particular colour states, we decompose the quark-gluon scattering amplitudes according to the irreducible colour representations, $\underline{3}\otimes\underline{8} = \underline{3}\oplus\underline{\bar{6}}\oplus\underline{15}$, exchanged in the u channel,

$$\mathcal{M} = \sum_{\chi} (P^{bb'})_{a'a}(\chi)\ \mathcal{M}_{\chi}\,, \tag{2.11}$$

where we recall that a and a' are quark colour indices and b and b' are gluon colour indices, and $\chi = \underline{3},\ \underline{\bar{6}},\ \underline{15}$. M_χ are colour-stripped coefficients and $(P^{bb'})_{a'a}(\chi)$ are the colour projectors,

$$\begin{aligned}
(P^{bb'})_{a'a}(\underline{3}) &= \frac{1}{C_F}(T^bT^{b'})_{a'a}\,,\\
(P^{bb'})_{a'a}(\underline{\bar{6}}) &= \frac{1}{2}\delta^{bb'}\delta_{a'a} - \frac{1}{N-1}(T^bT^{b'})_{a'a} - (T^{b'}T^b)_{a'a}\,,\\
(P^{bb'})_{a'a}(\underline{15}) &= \frac{1}{2}\delta^{bb'}\delta_{a'a} - \frac{1}{N+1}(T^bT^{b'})_{a'a} + (T^{b'}T^b)_{a'a}\,,
\end{aligned} \tag{2.12}$$

which fulfill the usual property of projectors,

$$(P^{bb'})_{a'a}(\chi)\ (P^{b'b''})_{aa''}(\chi') = \delta_{\chi\chi'}(P^{bb''})_{a'a''}(\chi)\,. \tag{2.13}$$

We find that

$$\mathcal{M}_{\underline{3}} = 2C_F\mathcal{A} - \frac{1}{N}\mathcal{B} + \mathcal{C}\,, \qquad \mathcal{M}_{\underline{\bar{6}}} = -\mathcal{B} + \mathcal{C}\,, \qquad \mathcal{M}_{\underline{15}} = \mathcal{B} + \mathcal{C}\,. \tag{2.14}$$

2.3 Signature of the exchanged state

In addition to having a particular colour, exchanged Reggeons must have a particular signature. In other words they are either even (positive signature) or odd (negative signature) under the exchange of s and t. We therefore define the amplitudes of specific signature to be,

$$\mathcal{M}^{\pm}_{\chi} = \frac{1}{2}\left(\mathcal{M}_\chi \pm \mathcal{M}_\chi(s\leftrightarrow t)\right). \tag{2.15}$$

Note that only the colour triplet positive signature exchange is expected to Reggeize. This is the contribution that will generate the LL and NLL behaviour of the amplitude. The other positive signature contributions and all of the negative signature contributions are not given by simple poles. In the LL approximation they can be obtained using the equation for amplitudes with fermion exchange derived in [11], but beyond the LL it is not known how to analyse these structures. Note also that $\mathcal{M}_\chi(s\leftrightarrow t)$ is in fact the amplitude for quark-antiquark annihilation into gluons.

As mentioned earlier, the colour structure $\delta^{bb'}\delta_{a'a}$ occurs first at one loop, while the Born contribution to $(T^{b'}T^b)_{a'a}$ vanishes in the Regge limit. Therefore, in the Regge limit, many of the amplitudes $\mathcal{M}^{(0)\pm}_\chi$ vanish in the Born approximation. In fact, from eqs. (2.2) and (2.3) we obtain,

$$\mathcal{M}^{(0)-}_{\underline{3}} = \mathcal{M}^{(0)+}_{\underline{\bar{6}}} = \mathcal{M}^{(0)-}_{\underline{\bar{6}}} = \mathcal{M}^{(0)+}_{\underline{15}} = \mathcal{M}^{(0)-}_{\underline{15}} = 0\,, \tag{2.16}$$

JHEP03(2002)032

and only $\mathcal{M}_{\underline{3}}^{(0)+}$ is non zero,

$$\mathcal{M}_{\underline{3}}^{(0)+} = 2C_F \mathcal{A}^{(0)} . \tag{2.17}$$

From eq. (2.6), for the one-loop coefficient of the positive signature triplet, we have

$$\begin{aligned} \mathcal{M}_{\underline{3}}^{(1)+} = \tilde{g}_S^2(u)\mathcal{M}_{\underline{3}}^{(0)+} \Bigg\{ \frac{C_F}{-\epsilon}\left[\ln\left(\frac{s}{-u}\right)+\ln\left(\frac{-s}{-u}\right)\right] - \frac{2(C_A+C_F)}{\epsilon^2} + \frac{3C_F}{\epsilon} + \\ + (\pi^2-7)C_F \Bigg\} + \mathcal{O}(\epsilon) , \end{aligned} \tag{2.18}$$

while for the higher-dimensional representations we have,

$$\mathcal{M}_{\underline{\bar{6}}}^{(1)+} = \mathcal{M}_{\underline{15}}^{(1)+} = 0 . \tag{2.19}$$

Thus, the colour representations $\underline{\bar{6}}$ and $\underline{15}$ are not present in the positive signature, at one-loop level and to leading power accuracy in s/u. In fact, in the LL approximation [11],

$$\mathcal{M}_{\underline{\bar{6}}}^{+} = \mathcal{M}_{\underline{15}}^{+} = 0 \tag{2.20}$$

to all orders.

For the one-loop coefficients of negative signature, we obtain,

$$\begin{aligned} \mathcal{M}_{\underline{3}}^{(1)-} &= \tilde{g}_S^2(u)\ \mathcal{M}_{\underline{3}}^{(0)+}\ \frac{1}{\epsilon}\left\{-C_F\left[\ln\left(\frac{s}{-u}\right)-\ln\left(\frac{-s}{-u}\right)\right]+\frac{1}{C_F}\ln\left(\frac{-s}{-t}\right)\right\} + \mathcal{O}(\epsilon) , \\ \mathcal{M}_{\underline{\bar{6}}}^{(1)-} &= \mathcal{M}_{\underline{15}}^{(1)-} = \tilde{g}_S^2(u)\ \mathcal{M}_{\underline{3}}^{(0)+}\ \frac{1}{\epsilon}\ \frac{1}{C_F}\ln\left(\frac{-s}{-t}\right) + \mathcal{O}(\epsilon) . \end{aligned} \tag{2.21}$$

Thus in the one-loop amplitude of negative signature all the three representations exchanged in the u channel contribute, to leading power accuracy in s/u.

3. Regge theory interpretation

Let's choose, for definiteness, the QCD Compton scattering process

$$q(p_a) + g(p_b) \longrightarrow q(p_{a'}) + g(p_{b'}) . \tag{3.1}$$

We will use physical polarizations of gluons, so that their polarization vectors satisfy $e(p_b) \cdot p_b = e(p_b) \cdot p_{b'} = 0$ and $e(p_{b'}) \cdot p_{b'} = e(p_{b'}) \cdot p_b = 0$. Then, for massless quarks, the contribution of the Reggeized quark to the amplitude for colour triplet exchange with even signature can be presented [11] as,

$$\mathcal{R}_{\underline{3}}^{+} = \Gamma_{\rm QG} \frac{-1}{\not{q}_\perp}\frac{1}{2}\left[\left(\frac{-s}{-u}\right)^{\delta_T(u)} + \left(\frac{s}{-u}\right)^{\delta_T(u)}\right]\Gamma_{\rm GQ} , \tag{3.2}$$

where $q_\perp$ is the transverse to the (p_a, p_b) plane part of the momentum transfer $q = p_{a'} - p_b$, $u = q^2 = q_\perp^2$ in the Regge limit, $\Gamma_{\rm QG}$ and $\Gamma_{\rm GQ}$ are the Reggeon vertices for the $G \to Q$ and $Q \to G$ transitions and δ_T determines the quark Regge trajectory. Note that for massless quarks δ_T depends only on $q_\perp^2$, so that instead of two complex conjugate trajectories with opposite parities for massive quarks we have a single one. We assume that the quark Regge trajectory has the perturbative expansion,

$$\delta_T(u) = \tilde{g}_S^2(u)\ \delta_T^{(1)} + \tilde{g}_S^4(u)\ \delta_T^{(2)} + \mathcal{O}(\tilde{g}_S^6(u))\,. \tag{3.3}$$

At leading order [11] and in $D = 4 + 2\epsilon$ dimensions,

$$\delta_T^{(1)} = -\frac{2C_F}{\epsilon}\,. \tag{3.4}$$

This is related to the analogous one-loop gluon trajectory by

$$\delta_T^{(1)} = \frac{C_F}{C_A}\omega^{(1)}.$$

The general structure of the Reggeon vertices is determined by relativistic invariance and colour symmetry. For the quark-gluon vertices the general structure is

$$\begin{aligned}\Gamma_{\mathrm{QG}} &= -g_S\bar{u}(p_{a'})T^b\left[\not{e}(p_b)(1+\delta_e(u)) + \frac{e(p_b)\cdot q\ \not{q}_\perp}{q_\perp^2}\delta_q(u)\right],\\ \Gamma_{\mathrm{GQ}} &= -g_S\left[\not{e}^*(p_{b'})(1+\delta_e(u)) + \frac{e^*(p_{b'})\cdot q\ \not{q}_\perp}{q_\perp^2}\delta_q(u)\right]T^{b'}u(p_a)\,.\end{aligned} \tag{3.5}$$

Note that it is straightforward to relate this general vertex structure to the scattering of particular particle helicities, section 2.1. The functions δ_e and δ_q represent the radiative corrections to the Born vertices. They are functions of $q_\perp^2$ in the massless case and have the perturbative expansion

$$\begin{aligned}\delta_e(u) &= \tilde{g}_S^2(u)\ \delta_e^{(1)} + \tilde{g}_S^4(u)\ \delta_e^{(2)} + \mathcal{O}(\tilde{g}_S^6)\,,\\ \delta_q(u) &= \tilde{g}_S^2(u)\ \delta_q^{(1)} + \tilde{g}_S^4(u)\ \delta_q^{(2)} + \mathcal{O}(\tilde{g}_S^6)\,.\end{aligned} \tag{3.6}$$

In the one-loop approximation the corrections were obtained in [27] and have the form

$$\begin{aligned}\delta_e^{(1)} &= \omega^{(1)}\left[\frac{C_F}{2C_A}\left(\frac{1}{\epsilon} - \frac{3(1-\epsilon)}{2(1+2\epsilon)} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon)\right) + \frac{1}{2\epsilon} - \right.\\ &\qquad\left. - \frac{\epsilon}{2(1+2\epsilon)}\right],\end{aligned} \tag{3.7}$$

$$\delta_q^{(1)} = \omega^{(1)}\frac{\epsilon}{2(1+2\epsilon)}\left(1 + \frac{1}{N^2}\right). \tag{3.8}$$

Note, that whereas $\delta_e^{(1)}$ has a soft $(1/\epsilon^2)$ singularity that must be cancelled by real radiation, $\delta_q^{(1)}$ is finite as $\epsilon \to 0$, since the corresponding spin structure is absent at leading order.

There is no ansatz for any of the odd signature exchanges or for the even signature $\underline{\bar{6}}$ and $\underline{15}$ exchanges. These contributions do not correspond to simple poles and cannot be described by an ansatz of the form of eq. (3.2). As mentioned earlier, in the LL approximation $\mathcal{M}^+_{\underline{\bar{6}}}$ and $\mathcal{M}^+_{\underline{15}}$ are zero to all orders. The LL negative signature contributions through to $\mathcal{O}(D-4)$ can be obtained from the equation derived in [11] and are given by

$$\begin{aligned}\mathcal{M}^-_{\underline{3}} &= \mathcal{M}^{(0)+}_{\underline{3}} \tilde{g}^2_S(u) \left(C_F + \frac{1}{C_F} \right) \frac{1}{\epsilon} \left(-i\pi + \tilde{g}^2_S(u) \frac{C_F}{\epsilon} \left(2i\pi \ln\left(\frac{s}{|u|}\right) + \pi^2 \right) + \right. \\ &\qquad \left. + \mathcal{O}\left(\tilde{g}^4_S(u)\right) \right), \end{aligned} \tag{3.9}$$

$$\begin{aligned}\mathcal{M}^-_{\underline{6}} &= \mathcal{M}^{(0)+}_{\underline{3}} \tilde{g}^2_S(u) \frac{1}{C_F \epsilon} \left(-i\pi + \tilde{g}^2_S(u) \frac{C_A + 2C_F - 1}{2\epsilon} \left(2i\pi \ln\left(\frac{s}{|u|}\right) + \pi^2 \right) + \right. \\ &\qquad \left. + \mathcal{O}\left(\tilde{g}^4_S(u)\right) \right), \end{aligned} \tag{3.10}$$

$$\begin{aligned}\mathcal{M}^-_{\underline{15}} &= \mathcal{M}^{(0)+}_{\underline{3}} \tilde{g}^2_S(u) \frac{1}{C_F \epsilon} \left(-i\pi + \tilde{g}^2_S(u) \frac{C_A + 2C_F + 1}{2\epsilon} \left(2i\pi \ln\left(\frac{s}{|u|}\right) + \pi^2 \right) + \right. \\ &\qquad \left. + \mathcal{O}\left(\tilde{g}^4_S(u)\right) \right). \end{aligned} \tag{3.11}$$

Note that at the one-loop level eqs. (2.20), (3.2)–(3.8) and (3.9)–(3.11) give all of the contributions to the scattering amplitude that survive in the Regge limit. They are in agreement with eqs. (2.6), (2.8), (2.18), (2.19), and (2.21) obtained from exact calculations.

3.1 Projection by tree-level amplitude

Let us denote the projection of the tree amplitude $\mathcal{M}^{(0)}$ summed over spins and colours on a generic amplitude $\mathcal{M}$ as,

$$\langle \mathcal{M}^{(0)} | \mathcal{M} \rangle = \sum_{\text{spin, col}} \mathcal{M}^{(0)^\dagger} \mathcal{M} \,. \tag{3.12}$$

For the Reggeized amplitude we obtain the projection normalised by the square of the Born amplitude to be,

$$\begin{aligned}\frac{\langle \mathcal{M}^{(0)} | \mathcal{R}^+_{\underline{3}} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} &= \exp\left(\delta_T(u)\, L\right) \frac{\left(1 + \exp^{-i\pi\delta_T(u)}\right)}{2} \left[(1 + \delta_e(u))^2 + \frac{(1+\delta_e(u))\delta_q(u)}{1+\epsilon} + \right. \\ &\qquad \left. + \frac{\delta^2_q(u)}{4(1+\epsilon)^2} \right], \end{aligned} \tag{3.13}$$

where $L = \ln(s/-u)$. Note that $\mathcal{R}^{(0)+}_{\underline{3}}$ coincides with the Born amplitude so that setting $\delta_e = \delta_T = \delta_q = 0$ produces unity.

Writing $\mathcal{R}^+_{\underline{3}}$ as a perturbative series,

$$\mathcal{R}^+_{\underline{3}} = \sum_n \tilde{g}^{2n}_S(u)\, \mathcal{R}^{(n)+}_{\underline{3}}, \tag{3.14}$$

and expanding eq. (3.13) to first order gives the one-loop contribution to the Reggeized amplitude $\mathcal{R}^{(1)+}_{\underline{3}}$ such that

$$\frac{\langle \mathcal{M}^{(0)} | \mathcal{R}^{(1)+}_{\underline{3}} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} = \delta^{(1)}_T\, L + 2\delta^{(1)}_e + \frac{\delta^{(1)}_q}{1+\epsilon} - i\frac{\pi}{2}\delta^{(1)}_T \,, \tag{3.15}$$

JHEP03(2002)032

that agrees with the results of [21, 22]. For the two-loop contribution we obtain

$$\frac{\langle \mathcal{M}^{(0)} | \mathcal{R}_{\underline{3}}^{(2)+} \rangle}{\langle \mathcal{M}^{(0)} \mathcal{M}^{(0)} \rangle} = \frac{(\delta^{(1)}_{T})^2}{2} L^2 + \left[\delta_T^{(2)} + \left(2\delta_e^{(1)} + \frac{\delta_q^{(1)}}{1+\epsilon} \right) \delta_T^{(1)} \right] L -$$
$$- \frac{\pi^2}{4} (\delta_T^{(1)})^2 + (\delta_e^{(1)})^2 + \frac{\delta_e^{(1)} \delta_q^{(1)}}{1+\epsilon} + \frac{(\delta_q^{(1)})^2}{4(1+\epsilon)^2} + 2\delta_e^{(2)} + \frac{\delta_q^{(2)}}{1+\epsilon} -$$
$$- i\frac{\pi}{2} \left((\delta_T^{(1)})^2 L + \frac{\delta_q^{(1)} \delta_T^{(1)}}{1+\epsilon} + 2\delta_e^{(1)} \delta_T^{(1)} + \delta_T^{(2)} \right) . \qquad (3.16)$$

Note that the Reggeized form for positive signature colour triplet exchange does not saturate the possible contributions to the cross section in the Regge limit. There is also a contribution from the negative signature exchange. This negative signature contribution appears first in the imaginary part of the (normalized) projection on the one-loop amplitude and also in the logarithmic imaginary part and non-logarithmic real parts of the two-loop amplitude.

Therefore, if the statement about the quark Reggeization is valid in the next-to-leading logarithmic order, the logarithmic terms in the right-hand side of eqs. (3.15) and (3.16) must coincide with corresponding terms for the total amplitude, which can be found using the results of [23] in the appropriate limit.

4. The Regge limit of the one- and two-loop calculations

The interference of the tree and one-loop amplitudes for quark-gluon scattering has been given in [21] while the interference of the tree and two-loop amplitudes has been computed in [23]. Taking the Regge limit or leading power of $s/|u|$ of these unrenormalised expansions, then we write,

$$\frac{\mathrm{Re}\langle \mathcal{M}^{(0)} | \mathcal{M}_{\underline{3}}^{(n)\pm} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} = \tilde{g}_S^{2n}(u) \sum_{m=0}^{n} B_{nm}^{\pm} \ln^m \left(\frac{s}{-u} \right) , \qquad (4.1)$$

$$\frac{\mathrm{Im}\langle \mathcal{M}^{(0)} | \mathcal{M}_{\underline{3}}^{(n)\pm} \rangle}{\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle} = -\frac{\pi}{2} \tilde{g}_S^{2n}(u) \sum_{m=0}^{n-1} D_{nm}^{\pm} \ln^m \left(\frac{s}{-u} \right) , \qquad (4.2)$$

where the positive and negative signature pieces are constructed according to eq. (2.15). For $n = 0$, $B_{00}^+ = 1$ and $B_{00}^- = 0$. In eq. (4.2), $n \geq 1$. In both eqs. (4.1) and (4.2) a sum over colours and helicities is implicit. For the interference of tree with one loop, the coefficients of the positive signature amplitudes are,

$$B_{11}^+ = -\frac{2}{\epsilon} C_F ,$$
$$B_{10}^+ = 2C_A \left(-\frac{1}{\epsilon^2} + \epsilon - 3\epsilon^2 \right) + 2C_F \left(-\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{(\pi^2-7)}{2} - (\zeta_3 - 6)\epsilon + \frac{(\pi^4 - 330)}{30} \epsilon^2 \right) ,$$
$$D_{10}^+ = -\frac{2C_F}{\epsilon} , \qquad (4.3)$$

JHEP03(2002)032

and those of negative signature are

$$B_{11}^{-} = 0\,, \qquad B_{10}^{-} = 0\,, \qquad D_{10}^{-} = \frac{2}{\epsilon C_F} + \frac{2C_F}{\epsilon}\,. \tag{4.4}$$

We immediately see that all of the positive signature contributions agree with the expansion of eq. (3.15) about $\epsilon = 0$. The leading and next-to-leading logarithmic terms B_{11}^{+} and B_{10}^{+} precisely match up while the corresponding negative signature terms are not present. The negative signature contribution is purely imaginary in this order. It is determined by the LL approximation and is in accordance with eq. (3.9).

At the two-loop level, the amplitude for quark-gluon scattering is not known as such, but it has been computed at the level of the interference with the tree amplitude [23], using conventional dimensional regularization (CDR) and renormalised in the $\overline{\mathrm{MS}}$ scheme. In ref. [23] the divergent contribution is written in terms of the infrared singularity operators $\mathbf{I}^{(1)}$, $\mathbf{I}^{(2)}$ and $\mathbf{H}^{(2)}$ introduced by Catani [28] and the tree- and one-loop amplitudes. The finite remainder is given in terms of logarithms and polylogarithms with arguments $-u/s$, $-t/s$ and u/t. Making the same expansion in the high energy limit and keeping only the leading power of $s/|u|$, we can extract the two-loop coefficients of the positive signature amplitudes,

$$\begin{aligned}
B_{22}^{+} &= \frac{1}{2}\left(B_{11}^{+}\right)^2, && (4.5)\\
B_{21}^{+} &= B_{11}^{+}B_{10}^{+} + C_F\beta_0\frac{2}{\epsilon^2} - C_F K\frac{2}{\epsilon} + C_F C_A\left(\frac{404}{27} - 2\zeta_3\right) + C_F N_F\left(-\frac{56}{27}\right) + \\
&\quad + C_F(C_F - C_A)\,(16\zeta_3)\,, && (4.6)\\
B_{20}^{+} &= \frac{1}{2}\left(B_{10}^{+}\right)^2 + C_A\beta_0\frac{1}{\epsilon^3} + C_F\beta_0\frac{1}{\epsilon^3} + C_A^2\left(\left(-\frac{67}{18} + \frac{\pi^2}{6}\right)\frac{1}{\epsilon^2} - \left(-\frac{193}{27} + \frac{11\pi^2}{18} + \zeta_3\right)\frac{1}{\epsilon} + \right.\\
&\qquad\left. + \left(-\frac{1736}{81} + \frac{67\pi^2}{54} + 4\zeta_3 + \frac{\pi^4}{12}\right)\right) + \\
&\quad + C_A C_F\left(\left(-\frac{83}{9} + \frac{\pi^2}{6}\right)\frac{1}{\epsilon^2} - \left(-\frac{3733}{108} + \frac{55\pi^2}{18} + 13\zeta_3\right)\frac{1}{\epsilon} + \right.\\
&\qquad\left. + \left(-\frac{71929}{648} + \frac{353\pi^2}{54} + \frac{242}{3}\zeta_3 + \frac{7\pi^4}{60}\right)\right) + \\
&\quad + C_F^2\left(-\left(-\frac{3}{4} + \pi^2 - 12\zeta_3\right)\frac{1}{\epsilon} + \left(-\frac{9}{8} + 2\pi^2 - 42\zeta_3 - \frac{28\pi^4}{45}\right)\right) + \\
&\quad + C_A N_F\left(\frac{5}{9\epsilon^2} - \left(\frac{19}{27} - \frac{\pi^2}{9}\right)\frac{1}{\epsilon} + \left(\frac{173}{81} - \frac{5\pi^2}{27} - 4\zeta_3\right)\right) + \\
&\quad + C_F N_F\left(\frac{14}{9\epsilon^2} - \left(\frac{317}{54} - \frac{5\pi^2}{9}\right)\frac{1}{\epsilon} + \left(\frac{5993}{324} - \frac{25\pi^2}{27} - \frac{8}{3}\zeta_3\right)\right) + \\
&\quad + \frac{C_A}{C_F}\left(-\frac{\pi^2}{\epsilon^2}\right) - (C_F - C_A)^2\frac{\pi^2}{\epsilon^2}\,, && (4.7)\\
D_{21}^{+} &= \left(B_{11}^{+}\right)^2, && (4.8)\\
D_{20}^{+} &= B_{21}^{+}\,, && (4.9)
\end{aligned}$$

where

$$\beta_0 = \frac{(11C_A - 2N_F)}{6}\,, \qquad K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} N_F\,. \tag{4.10}$$

Comparing the leading logarithmic contribution B_{22}^{+} (and D_{21}^{+}) with eq. (3.16) we immediately see that it is exactly as predicted — and is nothing more than a confirmation of the exponentiation of the leading logarithms. The next-to-leading logarithmic term B_{21}^{+} contains two pieces — one which is an echo of the one-loop coefficient function and the one-loop quark trajectory $B_{10}^{+}B_{11}^{+}$, and the other is the two-loop quark trajectory $\delta_T^{(2)}$ which appears for the first time,

$$\delta_T^{(2)} = C_F\left[\beta_0\frac{2}{\epsilon^2} - K\frac{2}{\epsilon} + C_A\left(\frac{404}{27} - 2\zeta_3\right) + N_F\left(-\frac{56}{27}\right) + (C_F - C_A)\,(16\zeta_3)\right]. \tag{4.11}$$

Note that eq. (4.11) has the remarkable feature that by mapping $C_F \to C_A$, we obtain the two-loop gluon Regge trajectory. The full quark trajectory through to two-loop order is thus,

$$\begin{aligned} j_Q(u) \;=\; \frac{1}{2} + \tilde g_S^2(u)\,\frac{\omega^{(1)}}{N}C_F\left[1 + \tilde g_S^2(u)\,\frac{\omega^{(1)}}{2N}\left\{\beta_0 - K\epsilon + \left(\left(\frac{202}{27} - 9\zeta_3\right)N - \frac{28}{27}N_F + \right.\right.\right.\\ \left.\left.\left. + 8\zeta_3 C_F\right)\epsilon^2\right\}\right]. \end{aligned} \tag{4.12}$$

The non-logarithmic term in the real part, B_{20}^{+}, belongs to the next-to-next-to-leading logarirhmic contribution and is not expected to match up with the ansatz of eq. (3.2) because of possible Regge cut contribution; conversely, the non-logarithmic term in the imaginary part, D_{20}^{+}, is next-to-leading and should be given by the anzatz of eq. (3.2). Comparing to eq. (3.16) one can easily see that it is the case.

Similarly the negative signature coefficients are given by

$$\begin{aligned} B_{22}^{-} &= 0\,,\\ B_{21}^{-} &= 0\,,\\ B_{20}^{-} &= (C_F - C_A)^2\frac{\pi^2}{\epsilon^2}\,, \end{aligned} \tag{4.13}$$

$$\begin{aligned} D_{21}^{-} &= -\frac{4}{\epsilon^2} - \frac{4C_F^2}{\epsilon^2}\,,\\ D_{20}^{-} &= -\left(1 + \frac{1}{C_F^2}\right)B_{21}^{+} + \frac{C_A}{C_F}\left(10 - 4\pi^2 - 8\zeta_3\right) + \frac{N_F}{C_F}12 + \\ &\quad + C_F^2\left(-8 + 8\pi^2 + 16\zeta_3\right) + C_A^2\left(-20 + 8\pi^2 + 16\zeta_3\right) + \\ &\quad + C_A C_F\left(44 - 20\pi^2 - 40\zeta_3\right) + 72C_F N_F - 36C_A N_F\,. \end{aligned} \tag{4.14}$$

The coefficients B_{22}^{-} and B_{21}^{-} vanish simply due to signature; B_{20}^{-} and D_{21}^{-} are determined by the LL approximation and are in accordance with eq. (3.9). It is not presently known how to interpret the coefficient D_{20}^{-}.

5. Conclusions

By taking the high-energy limit of the two-loop amplitudes for quark-gluon scattering [23], we have tested the validity of the general form of the high-energy amplitudes (3.2) for quark-gluon scattering, arising from a Reggeized quark with colour triplet and even signature exchanged in the crossed channel. The limit is compatible with the Reggeization in the leading and the next-to-leading orders. We have therefore extracted two-loop Regge trajectory for the quark, eqs. (4.11) and (4.12), as an expansion in powers of $(D-4)$ for the space-time dimension D tending to the physical value $D=4$. At present it is not known how to describe either the next-to-next-to-leading logarithmic triplet exchange contributions for either the positive signature B_{20}^{+} (given in eq. (4.7)) or the negative signature D_{20}^{-} (given in eq. (4.14)).

Acknowledgments

We thank Stefano Catani and Zoltan Trocsanyi for helpful comments. VDD thanks the IPPP and the CERN Theory Division for their kind hospitality during the early stage of this work. This work was supported in part by the EU Fourth Framework Programme "Training and Mobility of Researchers", Network "Quantum Chromodynamics and the Deep Structure of Elementary Particles", contract FMRX-CT98-0194 (DG-12-MIHT), and grants INTAS 97-31696, 00-00366 and RFBR 01-02-16042.

References

[1] M. Gell-Mann, M.L. Goldberger, F.E. Low, E. Marx and F. Zachariasen, *Elementary particles of conventional field theory as Regge poles, 3*, *Phys. Rev.* **B 133** (1964) 145.

[2] B.M. McCoy and T.T. Wu, *Theory of fermion exchange in massive quantum electrodynamics at high-energy, 1*, *Phys. Rev.* **D 13** (1976) 369.

[3] S. Mandelstam, *Non-Regge terms in the vector-spinor theory*, *Phys. Rev.* **137** (1965) B949.

[4] M.T. Grisaru, H.J. Schnitzer and H.-S. Tsao, *Reggeization of Yang-Mills gauge mesons in theories with a spontaneously broken symmetry*, *Phys. Rev. Lett.* **30** (1973) 811.

[5] M.T. Grisaru, H.J. Schnitzer and H.-S. Tsao, *Reggeization of elementary particles in renormalizable gauge theories-vectors and spinors*, *Phys. Rev.* **D 8** (1973) 4498.

[6] L.N. Lipatov, *Reggeization of the vector meson and the vacuum singularity in nonabelian gauge theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338.

[7] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, *On the Pomeranchuk singularity in asymptotically free theories*, *Phys. Lett.* **B 60** (1975) 50.

[8] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Multi-reggeon processes in the Yang-Mills theory*, *Zh. Eksp. Teor. Fiz.* **71** (1976) 840 [*Sov. Phys. JETP* **44** (1976) 443].

[9] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The Pomeranchuk singularity in nonabelian gauge theories*, *Zh. Eksp. Teor. Fiz.* **72** (1977) 377 [*Sov. Phys. JETP* **45** (1977) 199].

[10] I.I. Balitsky and L.N. Lipatov, *The Pomeranchuk singularity in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822.

JHEP03(2002)032

[11] V.S. Fadin and V.E. Sherman, *Fermion reggeization in non-abelian gauge theories*, *Zh. Eksp. Teor. Fiz.* **23** (1976) 599; *Processes with fermion exchange in nonabelian gauge theories*, *Zh. Eksp. Teor. Fiz.* **72** (1977) 1640 [*Sov. J. Nucl. Phys.* **45** (1977) 861].

[12] V.S. Fadin and R. Fiore, *The generalized nonforward BFKL equation and the 'bootstrap' condition for the gluon reggeization in the NLLA*, *Phys. Lett.* **B 440** (1998) 359 [hep-ph/9807472].

[13] V.S. Fadin and L.N. Lipatov, *BFKL pomeron in the next-to-leading approximation*, *Phys. Lett.* **B 429** (1998) 127 [hep-ph/9802290].

[14] G. Camici and M. Ciafaloni, *Irreducible part of the next-to-leading BFKL kernel*, *Phys. Lett.* **B 412** (1997) 396 [hep-ph/9707390], erratum *ibid.* **B417** (1998) 390.

[15] M. Ciafaloni and G. Camici, *Energy scale(s) and next-to-leading BFKL equation*, *Phys. Lett.* **B 430** (1998) 349 [hep-ph/9803389].

[16] V.S. Fadin, M.I. Kotsky and R. Fiore, *Gluon reggeization in QCD in the next-to-leading order*, *Phys. Lett.* **B 359** (1995) 181.

[17] V.S. Fadin, R. Fiore and M.I. Kotsky, *Gluon Regge trajectory in the two-loop approximation*, *Phys. Lett.* **B 387** (1996) 593 [hep-ph/9605357].

[18] V. Fadin, R. Fiore and A. Quartarolo, *Reggeization of quark-quark scattering amplitude in QCD*, *Phys. Rev.* **D 53** (1996) 2729 [hep-ph/9506432].

[19] J. Blumlein, V. Ravindran and W.L. van Neerven, *On the gluon Regge trajectory in $O(\alpha_s^2)$*, *Phys. Rev.* **D 58** (1998) 091502 [hep-ph/9806357].

[20] V. Del Duca and E.W.N. Glover, *The high energy limit of QCD at two loops*, *J. High Energy Phys.* **10** (2001) 035 [hep-ph/0109028].

[21] R.K. Ellis and J.C. Sexton, *QCD radiative corrections to parton parton scattering*, *Nucl. Phys.* **B 269** (1986) 445.

[22] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop helicity amplitudes for all $2 \to 2$ processes in QCD and $N = 1$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 411** (1994) 397 [hep-ph/9305239].

[23] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to massless quark gluon scattering*, *Nucl. Phys.* **B 605** (2001) 486 [hep-ph/0101304].

[24] M.L. Mangano and S.J. Parke, *Multiparton amplitudes in gauge theories*, *Phys. Rept.* **200** (1991) 301.

[25] S.J. Parke and T.R. Taylor, *An amplitude for n gluon scattering*, *Phys. Rev. Lett.* **56** (1986) 2459.

[26] V. Del Duca, A. Frizzo and F. Maltoni, *Factorization of tree QCD amplitudes in the high-energy limit and in the collinear limit*, *Nucl. Phys.* **B 568** (2000) 211 [hep-ph/9909464].

[27] V.S. Fadin and R. Fiore, *Calculation of reggeon vertices in QCD*, *Phys. Rev.* **D 64** (2001) 114012 [hep-ph/0107010].

[28] S. Catani, *The singular behaviour of QCD amplitudes at two-loop order*, *Phys. Lett.* **B 427** (1998) 161 [hep-ph/9802439].